6. THE THEORY OF APERTURE AERIALS

In aperture aerials electromagnetic waves are emitted trough the aperture. It separates the internal volume of the aperture aerial from the external space and presents a plane or a surface, which is transparent for electromagnetic waves. The process of electromagnetic waves formation is carried out by a weakly directional field source, which is placed in the internal volume of the aperture aerial and converts energy of high frequency currents into the electromagnetic waves energy. Formation of a field structure in the environmental space is completely defined by aperture parameters and distribution of tangential components of field in the aperture. As DD of aperture aerials as a matter of fact is created due to diffraction of electromagnetic waves on an aperture, such aerials sometimes are called diffraction antennas.

Irrespective of the constructive realization of the aperture aerial, it's radiation field is defined only by the aperture properties, that allows to examine such properties from common positions, namely, from position of influence of the tangential components' distribution in the aperture on the radiation field. Thus the aperture of the aerial can be analyzed as a system with continuous distribution of field sources, that enables to represent DC of aperture as the product of the elementary radiator DC and the array factor.

6.1. Analysis of a radiation field of aperture aerials

At approximate method of analysis of a radiation field the aperture is considered as a set of the Huygens elements. Such approach allows to determine the radiation field from the aperture as the sum of elementary radiators' fields. Therewith the distribution of tangential components in an aperture plane is considered given. Thus, in this method, here in after named the aperture method, the radiation field of the aerial system with continuous distribution of field sources is considered.

In Fig. 6.1 the flat aperture S corresponds to the xoy plane of rectangular coordinate system. Let us choose the observation point M in a far-field, with coordinates in spherical system $r,\ \theta\ and\ \varphi$.

E_S is the tangential field component in the aperture plane. Its module and

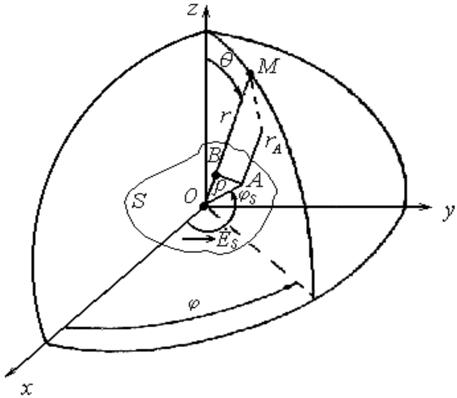

Fig. 6.1

phase depend on coordinates of aperture S. The elementary radiator, allocated in an aperture point A, in the observation point creates the field with the intensity

$$d\dot{E} = i\frac{\dot{E}_S dS}{2\lambda r} f_e(\theta,\varphi) e^{-ikr_A}, \qquad (6.1)$$

where dS is the area of the Huygens element; $f_e(\theta,\varphi)$ is DC of the Huygens element.

As point M is in the far-field, distance from point A to the observation point can be expressed through the distance from the coordinate origin and the path-length difference $OB = \Delta r$:

$$r_A = r - \Delta r. \qquad (6.2)$$

In Fig. 6.1 point B can be found as crossing point of direction r with a perpendicular, dropped from point A. The field of elementary radiators set, forming aperture S, is

$$\dot{E} = \int_S d\dot{E}. \qquad (6.3)$$

Substituting value of the field intensity of an elementary radiator (6.1) in formula (6.3) and taking into account expression (6.2)

$$\dot{E} = i\frac{f_e(\theta,\varphi)}{2\lambda}\frac{e^{-ikr}}{r}\int_S \dot{E}_S e^{-ik\Delta r} dS\,. \qquad (6.4)$$

The path-length difference Δr is determined by coordinates of the observation point M and by coordinates of point A (the Huygens element). If to use the scalar product of vectors

$$\Delta r = (r_0 OA), \qquad (6.5)$$

where r_0 is the unit vector, which coincides with the direction of radius vector r; OA is the vector, which is directed from the coordinate origin to point A and which module is equal to length OA .

In rectangular coordinate system vector r_0 is determined by the equation

$$r_0 = x_0 \sin\theta \cos\varphi + y_0 \sin\theta \sin\varphi + z_0 \cos\theta$$

and vector OA

$$OA = x_0 x + y_0 y\,,$$

where x *and* y are coordinates of point A .

From formula (6.5) the path-length difference is found:

$$\Delta r = x \sin\theta \cos\varphi + y \sin\theta \sin\varphi\,. \qquad (6.6)$$

When it is convenient to use the polar coordinate system in an aperture plane, point A is defined by the radius $\rho = OA$ and angle φ_S . Connection between polar and rectangular coordinates is expressed by formulas: $x = \rho \cos\varphi_S$ and $y = \rho \sin\varphi_S$.

Substituting these expressions in formula (6.6), we can receive the path-length difference as functions of coordinates in spherical and polar systems:

$$\Delta r = \rho \sin\theta \cos(\varphi - \varphi_S)\,. \qquad (6.7)$$

Taking into account expression (6.6), formula (6.4) is written down as

$$\dot{E} = i\frac{f_e(\theta,\varphi)}{2\lambda}\frac{e^{-ikr}}{r}\int_{a_1}^{a_2}\int_{b_1}^{b_2} \dot{E}_S e^{ik\sin\theta(x\cos\varphi + y\sin\varphi)} dxdy\,,$$

(6.8)

where $dS = dxdy,\ a_1 \le x \le a_2\ \ i\ \ b_1 \le y \le b_2$.

Let us find the aperture DF in the direction of axis z. In this case the path-length difference turns to zero $(\Delta r = 0)$. Power, which is radiated through the aperture

$$P_{\Sigma} = \int_S \Pi_S dS,$$

where Π_S is the power density in the aperture plane (integration will be carried out over the aperture area).

Assuming that the wave resistance of the aperture is equal to W_a, we obtain:

$$P_{\Sigma} = \frac{1}{2W_a} \int_S E_S^2 dS. \qquad (6.9)$$

Thus, as it follows from formulas (3.21) and (3.22), the DC value for the Huygens elements in the Z direction will be $1 + W/W_a$. Therefore the maximal value of the field intensity, radiated by the aperture in the specified direction, from formula (6.4) is

$$E_{max} = \frac{1}{2\lambda r}\left(1 + \frac{W}{W_a}\right)\left|\int_S \dot{E}_S dS\right|. \qquad (6.10)$$

Substituting expressions (6.9) and (6.10) in formula (2.27) at $F(\theta, \varphi) = 1$ the formula for calculation of the aperture DF can be found:

$$D = \frac{4\pi}{\lambda^2} \frac{(W + W_a)^2}{4W W_a} \frac{\left|\int_S \dot{E}_S dS\right|^2}{\int_S E_S^2 dS}. \qquad (6.11)$$

The effective area is determined by comparing expressions (6.11) and (2.37)

$$S_e = \frac{(W + W_a)^2}{4W W_a} \frac{\left|\int_S \dot{E}_S dS\right|^2}{\int_S E_S^2 dS}. \qquad (6.12)$$

As it follows from expression (6.12) the effective area at radiation in the Z direction depends on amplitude and phase distributions of tangential field components in the aperture area.

At $W_a = W$ the expression for the effective area can be simplified:

$$S_e = \frac{\left| \int\limits_S \dot{E}_S dS \right|^2}{\int\limits_S E_S^2 dS}. \qquad (6.13)$$

Formula (6.13) takes into account only the form of distribution of tangential field components in the aperture area.

6.2. The rectangular aperture with the uniform amplitude-phase distribution of tangential components

Let us consider the rectangular aperture with sizes a and b (Fig. 6.2), in which tangential components of field intensity vectors are

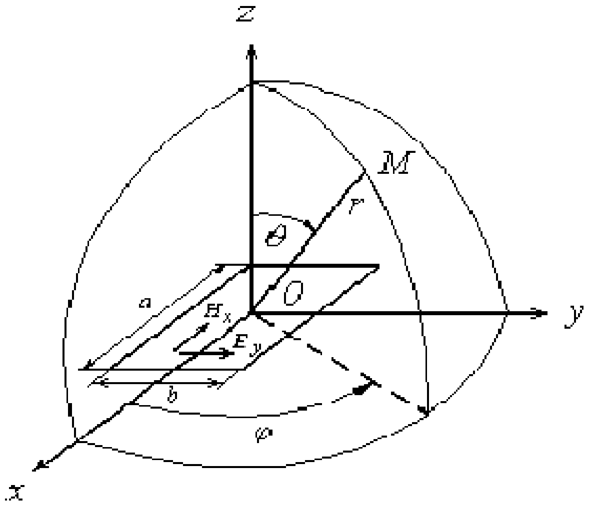

Fig. 6.2

equally directed at every point, and their amplitudes and phases do not depend on aperture coordinates.

Then

$$\dot{E}_S = E_y = E_t$$

and expression (6.8) takes the form

$$\dot{E} = i\frac{E_t}{2\lambda r} f_e(\theta,\varphi) e^{-ikr} \int\limits_{-a/2}^{a/2} \int\limits_{-b/2}^{b/2} e^{ik\sin\theta(x\cos\varphi + y\sin\varphi)} dxdy .$$

(6. 14)

Substituting DC of the Huygens element (3.21) and $\varphi = \pi/2$ in (6.14), we can find the field intensity in plane yoz (plane E):

$$\dot{E} = i\frac{E_t}{2\lambda r}\left(1 + \frac{W}{W_a}\cos\theta^E\right) e^{-ikr} \int\limits_{-a/2}^{a/2} \int\limits_{-b/2}^{b/2} e^{iky\sin\theta^E} dxdy .$$

After integration

$$\dot{E} = i\frac{E_t ab}{2\lambda r}\left(1 + \frac{W}{W_a}\cos\theta^E\right) \frac{\sin\left(\frac{kb}{2}\sin\theta^E\right)}{\frac{kb}{2}\sin\theta^E} e^{-ikr} . \qquad (6.15)$$

By analogy the field distribution in plane H (plane xoz) can be found:

$$\dot{E} = i\frac{E_t ab}{2\lambda r}\left(\frac{W}{W_a} + \cos\theta^H\right) \frac{\sin\left(\frac{kb}{2}\sin\theta^H\right)}{\frac{kb}{2}\sin\theta^H} e^{-ikr} . \qquad (6.16)$$

As is seen from formulas (6.15) and (6.16), the aperture DC is the product of the Huygens element DC and the array factor, which is the function $\sin u/u$, where u is the generalized angular argument. It is obvious, that $u = 0.5kb\sin\theta^E$.

The obtained expression for the array factor is identical to the array factor of the equal amplitude excitation and the equally spaced linear cophased array at a great number of radiators and distance between them, which satisfy the condition $d < \lambda$. Therefore the results of the following analysis within reasonable limits can be applied to the aerial array.

As is evident from (6.15) and (6.16), the rectangular aperture with the uniform amplitude-phase distribution of tangential components creates the maximal value of the field intensity in the normal direction (along axis z). In this case $\theta^E = \theta^H = 0$ and

$$E_{max} = \frac{E_t ab}{2\lambda r}\left(1 + \frac{W}{W_a}\right).$$

At changing coordinate angles θ^E and θ^H the directional characteristic of the Huygens element varies slower in comparison with function $\sin u/u$. Therefore the directional properties of the rectangular aperture are defined by function $\sin u/u$, which is plotted in Fig. 6.3.

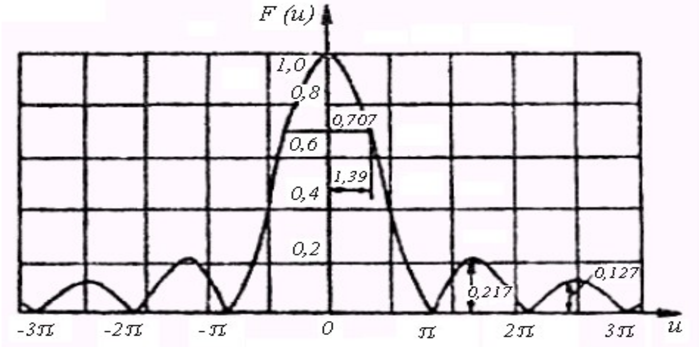

Fig. 6.3

This function reaches its maximal value at $u = 0$, takes zero values at $u = p\pi$, where $p = \pm 1, \pm 2, \pm 3$. From the conditions $0.5kb \sin\theta_{0p}^E = p\pi$ and $0.5ka \sin\theta_{0p}^H = p\pi$ the directions of zero radiation can be found:

$$\sin\theta_{0p}^E = \frac{p\lambda}{b}; \qquad \sin\theta_{0p}^H = \frac{p\lambda}{a}.$$

For the aperture of relatively large dimensions at $p = 1$

$$2\theta_0^E \approx 115^\circ \frac{\lambda}{b}; \qquad 2\theta_0^H \approx 115^\circ \frac{\lambda}{a}. \qquad (6.17)$$

The half-power beamwidth

$$2\theta_{0.5}^E \approx 51^\circ \frac{\lambda}{b}; \qquad 2\theta_{0.5}^H \approx 51^\circ \frac{\lambda}{a}. \qquad (6.18)$$

It follows from expressions (6.17) and (6.18), that the beamwidth in a considered plane is determined by the aperture size in the same plane and does not depend on the size of the aperture in other plane.

Directions of side lobes maximums are determined from the function $sin\, u/u$ extremums, which are at

$$u_{mp} = tg\, u_{mp} .$$

Solution of this equation for the first lobe maximum gives value of the generalized angular argument $u_{m1} = 1.4029\pi$.

Let us find the aperture effective area by formula (6.13). As $\dot{E}_S = E_t$, then

$$S_e = S , \qquad (6.19)$$

i.e., the effective area at the uniform amplitude and the phase distribution of tangential field components in the aperture plane is equal to the geometrical area.

The area utilization factor, as it follows from (2.38), is equal to unity and DF is determined from expression (2.37)

$$D = \frac{4\pi}{\lambda^2} S . \qquad (6.20)$$

Formulas (6.19) and (6.20) are obtained assuming that $W_a = W$.

6.3. Radiation from the circular aperture with cophased and uniform distribution of tangential components

The circular aperture is used in conical horn, parabolic, some lens and other aerials. In each point of the aperture with the uniform field distribution the tangential components of the electric intensity and magnetic intensity are characterized by the same amplitude and the same phase.

In Fig. 6.4 the circular aperture of radius a is presented. In the polar system position of an aperture element is described by coordinates: radius ρ and angle φ_S . Its area

$$dS = \rho d\rho d\varphi_S . \qquad (6.21)$$

Substituting DC of the aperture element (3.23), the path-length difference (6.7) and the element area (6.21) in expression (6.4),

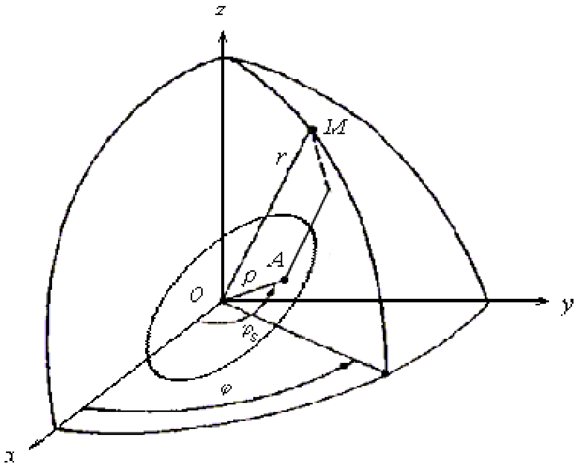

Fig. 6.4

the radiation field intensity of the circular aperture at $\dot{E}_S = E_t = const$ can be received:

$$\dot{E} = i\frac{E_t}{2\lambda r}(1+\cos\theta)e^{-ikr}\int\limits_{\rho=0}^{a}\int\limits_{\varphi_S=0}^{2\pi} e^{ik\rho\sin\theta\cos(\varphi-\varphi_S)}\rho\, d\rho d\varphi_S .$$

(6.22)

The received integral can be expressed through Bessel's function

$$\dot{E} = i\frac{E_t\pi a^2}{2\lambda r}(1+\cos\theta)\frac{2J_1(ka\sin\theta)}{ka\sin\theta}e^{-ikr},$$

where $J_1(u)$ is Bessel's function of the first order from the generalized angular coordinate: $u = ka\sin\theta$.

The directional characteristic, as well as for the rectangular aperture, is the product of two factors. At rather large apertures the array factor is

$$F(\theta) = \frac{2J_1(ka\sin\theta)}{ka\sin\theta}. \qquad (6.23)$$

Graphic representation of dependence (6.23) by its form is close to the curve, represented in Fig. 6.3. The beamwidth on the zero level from expression (6.23) is equal to

$$2\theta_0 \approx 140^\circ \frac{\lambda}{2a}.$$

The half-power beamwidth

$$2\theta_{0.5} \approx 58.5^\circ \frac{\lambda}{2a}.$$

The effective area as well as in the case of the rectangular aperture with the uniform distribution is equal to the geometrical area.

6.4. The aperture with non-uniform amplitude distribution

Let us consider the nonuniform distribution, which is frequently met in the aerial engineering, namely, the cosine amplitude distribution. The cosine amplitude distribution of the field intensity in the plane of the rectangular aperture (see Fig. 6.2) can be reduced to the case of radiation from the open end of a rectangular wave guide.

Let us assume, that the phase of the field intensity in all points of the aperture is the same. The amplitude of the field intensity along the directions, parallel to axis oy, does not change, and along the directions parallel to axis *ox*, obeys to the dependence

$$E_y = E_m cos\left(\frac{\pi x}{a}\right) \quad at \quad -\frac{a}{2} \le x \le \frac{a}{2}, \qquad (6.24)$$

where E_m is the electric field intensity in the aperture points, which are on axis oy.

Let us substitute values E_y in expression (6.8). DC of the aperture element is described by formula (3.22).

As a result of integration in borders of a variable x from $-a/2$ to $a/2$, variable y from $-b/2$ to $b/2$ we can receive the expression for the field intensity in plane *H,* which differs from expression (6.16):

$$\dot{E} = i\frac{\pi E_m ab}{4\lambda r}\left(\frac{W}{W_a} + \cos\theta^H\right)\frac{\cos\left(\frac{ka}{2}\sin\theta^H\right)}{\left(\frac{\pi}{2}\right)^2 - \left(\frac{ka}{2}\sin\theta^H\right)^2} e^{-ikr} .$$

In plane E, which coincides with plane yoz of the rectangular coordinate system (see Fig. 6.2), taking into account that the distribution of tangential components of the field intensity along axis oy is uniform, DC does not differ from expression (6.15).

In plane H normalized DC is determined by the formula

$$F(\theta^H) = \frac{\pi^2}{4}\frac{\frac{W}{W_a} + \cos\theta^H}{\frac{W}{W_a} + 1}\frac{\cos\left(\frac{ka}{2}\sin\theta^H\right)}{\left(\frac{\pi}{2}\right)^2 - \left(\frac{ka}{2}\sin\theta^H\right)^2} .$$

The array factor has significally changed in comparison with the corresponding array factor of the aperture with the uniform distribution (6.16). The factor takes the first zero value at

$$\frac{ka}{2}\sin\theta_0{}^H = \frac{3}{2}\pi .$$

Hence at rather big aperture sizes the beamwidth on zero radiation level in plane H is

$$2\theta_0{}^H \approx 172^\circ \frac{\lambda}{a} . \qquad (6.25)$$

The half-power beamwidth is determined by the expression

$$2\theta_{0.5}{}^H \approx 68^\circ \frac{\lambda}{a} . \qquad (6.26)$$

Comparing expressions (6.25) and (6.26) with expressions (6.17) and (6.18), one can note, that the DC major lobe in case of the cosine amplitude distribution is wider in comparison with the uniform distribution. At the uniform distribution of the field in the aperture the given beamwidth can be received with the smaller aperture sizes. Thus, owing to non-uniformity of excitation of the aperture, its geometrical area is used uneffectively. Aperture sites with small intensity values render small influence on the DD formation that is equivalent to reduction of the aperture area.

As the major lobe widens, the amplitude distribution falling down to the aperture edges, the intensity of side lobes increases. At the cosine distribution of tangential components the level of the first side lobe is $\nu_1 = -23$ dB, whereas at the uniform distribution $\nu_1 = -13.2$ dB. Examining other amplitude distributions, for example, cosine-on-pedestal, the following regularity can be noted: the faster the field intensity falls down to the aperture edges, the wider the major lobe and the more considerable the level of side lobes falls. Therefore apertures with the non-uniform amplitude distribution, which is chosen out of conditions of reduction of side lobes intensity up to a required level, are widely used in practice.

Data from Tab. 6.1 confirm the statement about the dependence of the DD beamwidth and the side lobes level upon the character of the amplitude distribution of intensity tangential components.

In Tab. 6.1, the function

$$u = \frac{ka}{2} \sin\theta$$

is used as the generalized angular argument.

The influence of velocity of the distribution reduction to the aperture edges is well illustrated by the cosine distribution in the first, the second and the third degrees. The higher the degree is, the faster the amplitude decreases, the wider DC is and the lower the level of the first side lobe is. In this case the area utilization factor also decreases.

Let us find by formula (6.13) the aperture effective area at the cosine amplitude distribution (6.24)

$$S_e = \frac{\left| \int\limits_{-a/2}^{a/2} \int\limits_{-b/2}^{b/2} E_m \cos\left(\frac{\pi x}{a}\right) dxdy \right|^2}{\int\limits_{-a/2}^{a/2} \int\limits_{-b/2}^{b/2} E_m^2 \cos^2\left(\frac{\pi x}{a}\right) dxdy}.$$

Table 6.1

Amplitude distribution, $f(x)$	Array factor, $F_\Sigma(u)$	Beamwidth		First side lobe level	Area utilization factor
		$2\theta_0$	$2\theta_{0.5}$	dB	
$const$	$\sin u/u$	$115^\circ\,\lambda/a$	$51^\circ\,\lambda/a$	-13,2	1
$\cos\left(\frac{\pi x}{a}\right)$	$\frac{\cos u}{(\pi/2)^2 - u^2}$	$172^\circ\,\lambda/a$	$67^\circ\,\lambda/a$	-23,3	0,81
$\cos^2\left(\frac{\pi x}{a}\right)$	$\frac{\sin u}{u(u^2 - \pi^2)}$	$229^\circ\,\lambda/a$	$83^\circ\,\lambda/a$	-32,0	0,667
$\cos^3\left(\frac{\pi x}{a}\right)$	$\frac{\cos u}{\left[u^2 - \left(\frac{\pi}{2}\right)^2\right]\left[u^2 - \left(\frac{3}{2}\pi\right)^2\right]}$	$287^\circ\,\lambda/a$	$95^\circ\,\lambda/a$	-40,0	0,575
$1 - 4(1-\delta)\frac{x^2}{a^2}$ at $\delta = 0.8$ $\delta = 0.5$ $\delta = 0$	$\frac{\sin u}{u} + (1-\delta)\frac{(2-u^2)\sin u}{u^3} - \frac{2(1-\delta)\cos u}{u^2}$	$121.5^\circ\,\lambda/a$ $130.5^\circ\,\lambda/a$ $164^\circ\,\lambda/a$	$52.7^\circ\,\lambda/a$ $55.5^\circ\,\lambda/a$ $66^\circ\,\lambda/a$	-15,8 -17,1 -20,6	0,994 0,97 0,833
$1 - \left\|\frac{2x}{a}\right\|$	$\left(\sin\frac{u}{2}\Big/\frac{u}{2}\right)^2$	$229^\circ\,\lambda/a$	$73^\circ\,\lambda/a$	-26,4	0,75